Inelastic x-ray scattering study of the collective dynamics in liquid sodium

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(Received 19 September 2001; published 7 March 2002)

Inelastic x-ray scattering data have been collected for liquid sodium at \( T = 390 \) K, i.e., slightly above the melting point. Owing to the very high instrumental resolution, pushed up to 1.5 meV, it has been possible to determine accurately the dynamic structure factor \( S(Q, \omega) \) in a wide wave-vector range, 1.5–15 nm\(^{-1}\), and to investigate on the dynamical processes underlying the collective dynamics. A detailed analysis of the line shape of \( S(Q, \omega) \), similarly to other liquid metals, reveals the coexistence of two different relaxation processes with slow and fast characteristic time scales. The present data lead to the conclusion that (i) the picture of the relaxation mechanism based on a simple viscoelastic model fails and (ii) although the comparison with other liquid metals reveals similar behavior, the data do not exhibit an exact scaling law as the principle of the corresponding state would predict.

DOI: 10.1103/PhysRevE.65.031205

PACS number(s): 61.20.–p, 67.55.Jd, 67.40.Fd, 61.10.Eq

I. INTRODUCTION

Liquid metals are well known to exhibit remarkably pronounced inelastic features (Brillouin peaks) in their density fluctuation spectra up to very high \( Q \) values (well beyond the hydrodynamic regime). For this main reason, in the last decades, the dynamic structure factor of these systems has been used as a benchmark for the comprehension of the mechanisms underlying the atomic motions at the microscopic level in liquids. In particular, in the special case of alkali metals, collective oscillatory modes persist down to wave-lengths of the order of one or two interparticle distances, a feature that makes them ideal candidates for testing different models for collective properties at finite wave vectors.

In the last 30 years many experimental investigations have been performed on these systems by means of inelastic neutron scattering (INS), which was, up to a few years ago, the only tool available for the investigation of the dynamics of condensed matter in the mesoscopic wave-vector region. Indeed, since the very first inelastic neutron scattering experiments by Copley and Rowe [1] in liquid rubidium, many similar studies have been reported for similar systems: lithium [2], sodium [3], potassium [4], cesium [5], and again rubidium [6]. In the most favorable cases (for example, in cesium), important features such as the so-called positive dispersion of sound, namely, an upwards deviation from the low-\( Q \), linear dispersion behavior have been reported. Moreover, a description of the coherent dynamic structure factor has been given in terms of a single relaxation process (viscoelastic model). On the other hand, in some of the above-mentioned systems, an accurate investigation of the collective properties is prevented by two drawbacks of the INS technique, namely, the presence of both coherent and incoherent contribution to the inelastic scattering cross section and the occurrence of kinematic restrictions, which confine the minimum accessible wave vector above a threshold value, which is already outside the linear dispersion regime. This is particularly the case in lighter elements such as lithium and sodium.

Despite the lack of experimental information on the collective properties in these systems, significant advances have been made by means of computer simulation techniques, where the aforementioned restrictions do not apply. Consequently, many numerical studies have been reported aiming at the study of the density fluctuation spectra of liquid metals [7–13]. Moreover, by these numerical techniques one can access a wider set of correlation functions, while an inelastic scattering experiment basically probes the density autocorrelation function.

On the theoretical side, thanks to the development of approaches such as the memory function formalism, the relaxation concept, and the kinetic theory, [14–24] a framework has been built up to be able to account for the behavior of the aforementioned correlation functions. Along this line, a general picture of the dynamics of the density fluctuations in simple liquids gradually emerged. In particular, it is now established that the decay of the density autocorrelation function occurs through mechanisms characterized by different time scales. In addition to a mechanism related to the coupling of the density and the temperature modes (thermal relaxation), one must allow for the dynamical effects of the stress correlation function (viscous relaxation). This latter process is thought to proceed throughout two distinct relaxation channels, active over quite different time scales: a rapid decay, customarily ascribed to the interaction of each atom and the “cage” of its neighboring atoms, is followed by a slower process that yields a long lasting tail. This mechanism stems from slow, temperature-dependent rearrangements that eventually, in those systems capable of sustaining supercooling, may cause a structural arrest (glass transition). This structural relaxation process has widely been studied within the mode coupling formalism. Recently, in the specific case of liquid alkali metals, remarkable attempts have been made to set up self-consistent approaches, which include all these mechanisms from the very start [22,25].

This theoretical framework has been tested on a number of numerical studies (see, for example, [14]). On the experimental side, however, the line shapes extracted by INS in
those systems where one may accept a significant \((Q - E)\)
region did not allow to discriminate between different models
including all the relevant relaxation processes [5].

Although through INS, enormous advances in the compre-
prehension of the collective properties of condensed matter
have been reported, some basic aspects remain still unsettled.
In particular, due to the previously mentioned features of
INS, sufficiently accurate measurements of the coherent dy-
namic structure factor \(S(Q, \omega)\) have hardly been reported,
with the result that a quantitative description of the relax-
ation dynamics in monoatomic fluids was mainly based on
simulation data.

Only in the recent past, the development of new synchro-
tron radiation facilities have opened the possibility of using x
rays to measure \(S(Q, \omega)\) in the nonhydrodynamic region; in
this case the photon speed is obviously much larger than the
velocity of the excitations and no kinematic restriction oc-
curs. Moreover, in a monatomic system, the x-ray scattering
cross section is purely coherent, and consequently can be
directly associated with the coherent dynamic structure fac-
tor [14,26]. After some early, pioneering data [27], inelastic
x-ray scattering (IXS) rapidly developed in the last decade,
becoming fully operational with the advent of the third-
generation sources [28,29]. The applicability of IXS to the
study of collective and single-particle dynamics of light liq-
uid metals has been then exploited in several works per-
formed at the beamline ID16 of the European synchrotron
radiation facility (ESRF) in Grenoble [30–32], thus showing
the complementarity of IXS and INS. More recently, IXS has
been pushed to the top of its theoretical performance in terms
of flux and resolution, and therefore new and more accurate
IXS studies on liquid metals allowed to explore the relaxation
processes underlying the microscopic dynamics in monoatomic fluids [33–35]. In particular, it has been possible
to detect experimentally and quantify the presence of the
relaxation scenario predicted by the theory of simple liq-
uids. In detail, the data have been analyzed within a gener-
alized hydrodynamic approach [14,15] and, following a
memory function formalism [16] in the mesoscopic wave-
vector region, it has been possible to detect the presence of
two viscous relaxation processes in addition to the thermal
process [34]. These results, obtained with a phenomenologi-
 cal ansatz for the memory function originally proposed in a
theoretical work by Levesque et al. [17], have then been
deply discussed in order to clarify the physical origin of all
these dynamical processes [33].

In this work, we report the results of a very high resolu-
tion determination of the dynamic structure factor in liquid
sodium at the melting point at several fixed \(Q\) values. Several
approximations are adopted for the analysis of the experi-
mental data, and the number and the role of the involved
relaxation processes is discussed in detail. Finally, a com-
parison with similar experimental data on different liquid
metals is presented.

II. THE EXPERIMENT

In liquid sodium, the coherent and incoherent INS cross
sections are almost equivalent, and the adiabatic speed of
sound is about 2500 m/s. As a result, a study of the collective
properties in the \(Q\) region characterized by a linear disper-
sion (the first Brillouin pseudophone) is quite hard to perform
[3]. The use of IXS in these systems is, therefore, particu-
larly advantageous and, as shown in Ref. [32], the inelastic
features of the dynamic structure factor can easily be de-
tected even around the \(Q\) position of the first sharp diffraction
peak \((Q_n \approx 20 \text{ nm}^{-1})\).

Data have been collected for liquid sodium at the new
IXS beamline ID28 of ESRF (which is now fully operative),
allowing an energy resolution of \(\Delta E = 1.5 \text{ meV FWHM}\).
The experiment has been performed at a fixed exchanged
wavevector over a \(Q\) region of 1.5–15 \text{ nm}^{-1}. Each energy
scan \((-50<E<50 \text{ meV})\) took about 300 min, and has been
repeated for a total integration time of about 500 s/point.
A five-analyzer bench, operating in horizontal scattering geom-
etry, allowed us to collect simultaneously photons at five
different values of exchanged wavevector \(Q\) for each single
scan. As a sample environment we used a setup similar to
that adopted in previous experiments performed on liquid
lithium [31], namely, an austenitic steel cell heated by ther-
mal contact with a resistor connected to a voltage regulated
supply. The sample length has been optimized by matching
the absorption length of sodium at the working energy of
21747 eV [corresponding to the Si \((111 111)\) reflection in
backscattering geometry], which is about 5 mm.

In Fig. 1, a selection of experimental spectra is reported.
Since the incident flux on the sample varies with time, data
have been normalized to the monitor. Moreover, since the
five analyzers have different scattering efficiencies, to extract
\(\bar{S}(Q, \omega)\) from the raw spectra, the data have been put in
absolute units using the knowledge of the lower order fre-
cquency momenta of the dynamic structure factor,

\[
\Omega_{S}^{(0)} = \int d\omega \bar{S}(Q, \omega) = \bar{S}(Q),
\]

\[
\Omega_{S}^{(1)} = \int d\omega \bar{S}(Q, \omega) \omega^2 = \frac{\hbar Q^2}{2M},
\]

where the tilde indicates the true quantum dynamic structure
factor, to distinguish it from its classical representation. This
procedure can be summarized as follows. The actual experi-
mental spectrum \(I(Q, \omega)\) is proportional to the convolu-
tion of the scattering law \(\bar{S}(Q, \omega)\) with the resolution \(R(\omega)\),

\[
I(Q, \omega) = \tilde{E}(Q) \int d\omega' \bar{S}(Q, \omega') R(\omega - \omega').
\]

In terms of the first two spectral moments of \(I\) and \(R\): \(\Omega_{I}^{(0)}\),
\(\Omega_{I}^{(1)}\), \(\Omega_{R}^{(0)}\), \(\Omega_{R}^{(1)}\), one easily deduces that

\[
\bar{S}(Q) = \frac{\hbar Q^2}{2M} \left( \Omega_{I}^{(1)}/\Omega_{I}^{(0)} - \Omega_{R}^{(1)}/\Omega_{R}^{(0)} \right)^{-1}.
\]

Consequently, the normalized spectrum (still affected by the
resolution broadening) reads
expressed in terms of the real and imaginary part of the complex function $M(Q,\omega)$, the Fourier-Laplace transform of $M(Q,t)$ as [14]

$$
S(Q,\omega) = \frac{\pi^{-1} \omega_0^2(Q) \hat{M}'(Q,\omega)}{\omega^2 - \omega_0^2(Q) + \omega \hat{M}''(Q,\omega)}
$$

where the quantity $\omega_0^2(Q) = KTQ^2/mS(Q)$ is related to the generalized isothermal sound speed through the relation $c_i(Q) = \omega_0^2(Q)/Q$.

In order to use the above expression to reproduce the experimental spectra, $I_N(Q,\omega)$, it has to be modified to be quantum compliant with the detailed balance condition, and finally must be convoluted with the instrumental resolution $R(\omega)$. Utilizing one of the most common quantum transformations, one finds that

$$
I_N^0(Q,\omega) = \int \frac{\hbar \omega'}{1 - e^{-\hbar \omega'/KT}} S(Q,\omega') R(\omega - \omega') d\omega'.
$$

The above expression can finally be used as a fitting function to the experimental data to obtain the relevant relaxation parameters, i.e., relaxation times and strengths. In this procedure, the values of $\hat{S}(Q)$ are extracted from the experimental data exploiting Eq. (4). As memory function $M(Q,t)$, we used a multiple exponential ansatz. In Ref. [34], indeed, it has been shown that in a similar system, namely, liquid lithium (i) at least three distinct relaxation processes (one thermal and two viscous) are necessary to reproduce the IXS line shape and (ii) a multiexponential shape is an acceptable approximation of the actual memory function. Consequently, the total memory function reads

$$
M(Q,t) = (\gamma - 1) \omega_0^2(Q) e^{-D_T Q^2} + \Delta^2(Q)[A(Q)e^{-t/\tau_v(Q)}
$$

$$
+ \{1 - A(Q)\} e^{-t/\tau_v(Q)}].
$$

The first term of the above equation comes from the coupling between thermal and density degrees of freedom, and depends on the thermal conductivity $D_T$ and the specific heat ratio $\gamma$. The remaining two contributions in Eq. (8) are the genuine viscous processes. Through the fitting procedure, the unknown parameters $\tau_v, \tau_a$ (the relaxation times), $A(Q)$ (the relative weight of the viscous processes), and $\Delta^2(Q)$ (the total viscous strength) can be deduced.

In Fig. 2, we report the outcome of this fitting procedure, i.e., a comparison between $I_N^0(Q,\omega)$ and $I_N(Q,\omega)$. Both the results of a one- and a two-time ansatz are shown. As in the case of lithium and aluminum, a single relaxation time description (viscoelastic model) appears to yield a rather poor reproduction of the data. In particular, at low $Q$’s the simultaneous presence of two processes is crucial even to catch just the qualitative features of the spectra.

As previously mentioned, once the experimental values of $\hat{S}(Q)$ are introduced in Eq. (6), the only free fitting parameters are the relaxation times and strength. A consistency check has, however, been obtained comparing these values of $\hat{S}(Q)$ (sum rules) with that calculated from $\omega_0^2(Q)$ left as...
free parameter in Eq. (6). In Fig. 3 the results are shown, together with the experimental determination of $S(Q)$ from neutron diffraction data [36].

In Fig. 4 we show the two relaxation times $\tau_\alpha$ and $\tau_\mu$ as obtained by the fitting procedure. The slower process is almost constant, with an abrupt increase below $Q\approx 6$ nm$^{-1}$. Such an increase is probably an artifact, since it occurs as the time scale approaches the inverse of the resolution width, here reported as a dashed line. Consequently the determination of the $\alpha$ relaxation time below this $Q$ value is no longer reliable. The $\mu$ relaxation process shows instead a slightly decreasing $Q$ dependence. In the insets, we report $\omega \tau$ as function of $Q$ for the two processes, where $\omega$ is the dominant frequency of the density fluctuations at a given $Q$ (i.e., the frequency of the Brillouin component). The quantity $\omega \tau$ gives important indications about the role of the two processes. The faster one, indeed, has values of $\omega \tau$ always smaller than 1, so that it is mainly responsible for the acoustic damping (Brillouin linewidth). Moreover, $\omega(Q)\tau_\mu(Q)$
shows an increase in the same wave-vector region where the sound dispersion occurs (approximately, up to $Q$ values of one-half of the position of the first sharp diffraction peak), so that an increase of the apparent peak position ruled by the strength of the fast process has to be expected. The $\omega(Q)\tau_\alpha(Q)$ is always larger than 1. Consequently the $\alpha$ process mainly controls the sharper portion of the quasielastic scattering, while it only gives a moderate contribution to the speed of sound over the whole explored $Q$ range (the strength of the $\alpha$ process is negligible with respect to that of the fast relaxation).

The above-mentioned increase of the speed of sound is illustrated in Fig. 5, where $\omega_0(Q)$, the maximum of the longitudinal current correlation spectrum $J^\parallel(Q,\omega) = \omega^2/Q^2 S(Q,\omega)$, is reported in the main frame and the associated sound speed $\omega_0(Q)/Q$ is reported in the inset. The positive dispersion turns out to be of the order of 20%, similarly to what is found in other liquid metals [14]. In contrast with the results reported in Ref. [32], we find that the isothermal value of the sound speed is not yet reached at the minimum wave vector probed in our experiment, i.e., 1.5 nm$^{-1}$. At this low $Q$, resolution effects are particularly important: even with the present resolution (1.5 meV FWHM), the Brillouin components of the $S(Q,\omega)$ appear as shoulders of the quasielastic signal, therefore we believe that this discrepancy may be ascribed to the lower resolutions of the previous experiment, which prevents an accurate determination of the actual line shape at such a small $Q$ value. Both the isothermal $c_\omega(Q)$ (low frequency) [37] and the infinite frequency $c_\infty(Q)$ values of the sound speed have been reported as obtained by the fit. This latter quantity is always higher than the maximum value reached by the apparent sound speed, consistently with the behavior of the relaxation times: as shown in Fig. 4, the faster process (which is that with the dominant strength) has a relaxation time, which never exceeds the dominant time scale of the density fluctuation at the probed wave vector. This notwithstanding, the actual values of $c_\infty(Q)$ reported in Fig. 5 can partially be affected by the oversimplifications given by the assumption of exponential line shape for the memory functions [33].

We conclude this investigation with the comparison of the $S(Q,\omega)$ of three different liquid metals. In Fig. 6 the IXS data on liquid sodium are compared with those of lithium and aluminum by introducing a set of length, mass, and time units [$l^*,m^*,\tau^*$] for each system. Such a procedure can be considered meaningful as far as a corresponding state principle is valid. As length unit $l^*$, we have chosen $l^* = Q_m^{-1}$, the inverse of the position of the static structure factor $S(Q)$. The time unit $\tau^*$ has been instead chosen as $\tau^* = \sqrt{m/T_m l^*}$, where $T_m$ is the melting temperature and $m$ is the atomic mass. In the upper panel the raw IXS signal, scaled on these
units is reported. The $Q$ values have been selected among the available ones to be as close as possible for three different regions of the dispersion curve. It is worth pointing out that all the spectra are resolution broadened and, moreover, that the three experiments have been performed in different conditions of resolutions. To bypass the instrumental resolution bias, one can compare the line shape obtained by the fits, i.e., the classical deconvoluted dynamical structure factor that reproduces the experimental data better after the transformation of Eq. (7). The result of such a comparison is reported in the lower panel of Fig. 6. The aluminum spectra clearly have a different line shape, while lithium and sodium are much more similar when scaled in their respective absolute units. This notwithstanding, the line shapes of the two alkali systems do not coincide, implying that the principle of corresponding state is not entirely valid for these systems. One comes to the same conclusion even by looking at some basic quantities such as the sound speed: the scaling factors between lithium and sodium, for example, are not the same if one looks at $c_s(Q \rightarrow 0)$ or $c_v(Q \rightarrow 0)$. Also the static structure factor $S(Q)$, being of a rather universal quantity, depends on the considered system.

IV. CONCLUSIONS

In conclusion, we have presented a very high resolution experimental study of the collective dynamic structure factor in liquid sodium at the melting temperature. In order to account for all the spectral details, beside the usual thermal process coupling temperature and density, at least two additional viscous relaxation processes have to be invoked. The viscoelastic approximation does not give, indeed, a satisfactory description of our data. These results have finally been compared with those obtained in other liquid metals. Although the mechanisms ruling the dynamics turn out to lie in a general framework characterizing both alkali and nonalkali simple liquids, some important quantitative differences are found: in particular, a scaling law among alkali systems seems to be inapplicable, in contrast to some previous findings obtained by computer simulation data.

ACKNOWLEDGMENTS

We are thankful to the ESRF staff for the fine experimental conditions and for their assistance during the experiment. One of the authors (T.S.) acknowledges R. Yulmetyev and M. Silbert for stimulating discussions.


[37] Due to the high thermal conductivity of liquid metals as compared to that of ordinary fluids, the crossover in the sound speed associated with the thermal relaxation occurs well below 1 nm$^{-1}$, which is approximately the lower $Q$ accessible to IXS. Consequently, in the IXS window the low frequency limit of the sound speed is the isothermal rather than the adiabatic sound speed [33].